

Lecture No. 12



Collective effects. Single and Multibunch Instabilities

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Introduction



- **Charged particles in a beam mutually interact and interact with the conductive walls of the vacuum chamber.**
 - These effects are usually referred as **collective effects**.
 - Collective effects play a major role, quite often limiting the final performance of an accelerator.
- For example, we already saw that the Touschek effect can generate losses in electron storage rings. Other examples include space charge limiting the minimum emittance and the maximum current in proton storage rings, beam-beam effects reducing the luminosity performance in colliders, ...
- Additionally, collective effects make particles within the bunch and between bunches “communicate”, allowing for single bunch and multibunch instabilities.
- In designing high performance accelerators, collective effects need to be carefully taken into account and solutions for minimizing these effects need to be adopted.
- Solutions can be *passive*, when in the design phase the parameters are chosen in order to contain collective effects, or *active* where the accelerator operates above instability threshold but *feedback* systems damp the instabilities down.

Space-Charge



We already saw how Coulomb scattering between two particles in the beam can generate particle losses by the Touschek Effect.

But Coulomb interaction is also responsible for the so-called **space charge effect. In this case, the generic particle in the bunch experiences the *collective* Coulomb force due to the field generated by the charge of all the other particles in the bunch.**

Such fields, referred also as *self-fields*, are quite nonlinear and their evaluation usually requires numerical techniques.

Anyway, by using the proper approximation, it is possible to obtain analytical solutions that gives us some useful insights on the effect.

Teng Solution and the Gaussian case



- By assuming a *continuous* (non-bunched) beam with *constant linear charge density* λ and with a *stationary uniform elliptical* distribution in the transverse plane, Teng in 1960 found the following expression for the fields inside the beam:

$$E_x = \frac{1}{\pi\epsilon_0} \frac{\lambda}{a(a+b)} x, \quad E_y = \frac{1}{\pi\epsilon_0} \frac{\lambda}{b(a+b)} y, \quad B_x = -\frac{\mu_0}{\pi} \frac{\lambda\beta c}{b(a+b)} y, \quad B_y = \frac{\mu_0}{\pi} \frac{\lambda\beta c}{a(a+b)} x$$

with a and b the ellipse half-axes, and the beam moving along z with velocity βc .

- For a more realistic gaussian distribution in the transverse plane and for $x \ll \sigma_x$ and $y \ll \sigma_y$:

$$E_x = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{\sigma_x(\sigma_x + \sigma_y)} x, \quad E_y = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{\sigma_y(\sigma_x + \sigma_y)} y, \quad B_x = -\frac{\mu_0}{2\pi} \frac{\lambda\beta c}{\sigma_y(\sigma_x + \sigma_y)} y, \quad B_y = \frac{\mu_0}{2\pi} \frac{\lambda\beta c}{\sigma_x(\sigma_x + \sigma_y)} x$$

- For both cases the fields scale linearly with x and y , and:

$$B_x = -\frac{\beta}{c} E_y, \quad B_y = \frac{\beta}{c} E_x,$$

Effects of the Space-Charge



- Such space charge fields exert forces on the beam particles, whose intensities are given by the Lorentz Force:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

And using the
previous relations:

$$F_x = q(E_x - \beta c B_y) = qE_x(1 - \beta^2) \propto \lambda(1 - \beta^2)x$$

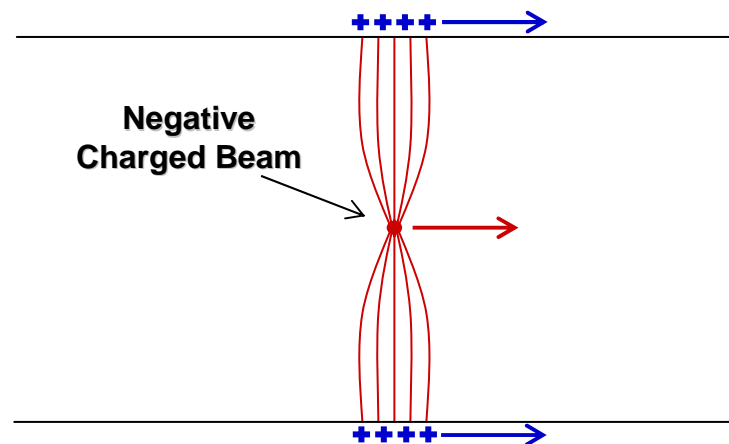
$$F_y = q(E_y + \beta c B_x) = qE_y(1 - \beta^2) \propto \lambda(1 - \beta^2)y$$

- The last equations show that the **space charge forces become negligible for relativistic beams**.
- They also show that for the non-relativistic beam, the forces are repulsive and proportional to the distance from the beam center.
- This is equivalent to a defocusing quadrupole in both planes with strength proportional to the current in the beam.
- Such a situation generates a **betatron tune shift with current** for the particles in the core of the beam.
- For the non-core particles the linear dependence of the force breaks and numerical calculations are required for the evaluation of the space charge effects.

Vacuum Chamber Effects: Image Charge



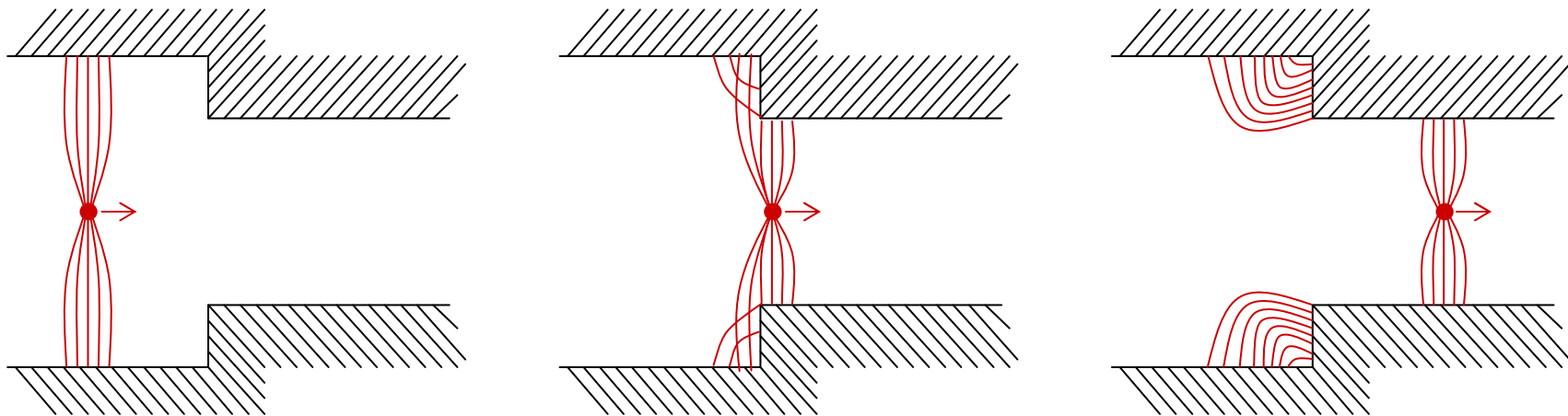
- In the lab system the beam electromagnetic field of a relativistic particle is transversely confined within an angle of $\sim 1/\gamma$ (where γ is the particle energy in rest mass units).
- Particle beams requires ultra high vacuum pressures, that can be achieved inside special metallic vessels called *vacuum chambers*.
- For the Maxwell equations, the electric field associated with the particle beam, must terminate perpendicularly on the chamber equipotential conductive walls.
- This boundary conditions requires that the same amount of charge but with opposite sign, travels on the vacuum chamber together with the beam. Such charge is referred as the **image charge**.



Vacuum Chamber Wake Fields



- The beam and its electromagnetic field travel inside the vacuum chamber while the image charge travels on the chamber itself.
- Any variation on the chamber profile, on the chamber material, or on the material properties breaks this configuration.
- The result is that the beam loses a (usually small) part of its energy that feeds the electromagnetic fields that remain after the passage of the beam. Such fields are referred to as **wake fields**.

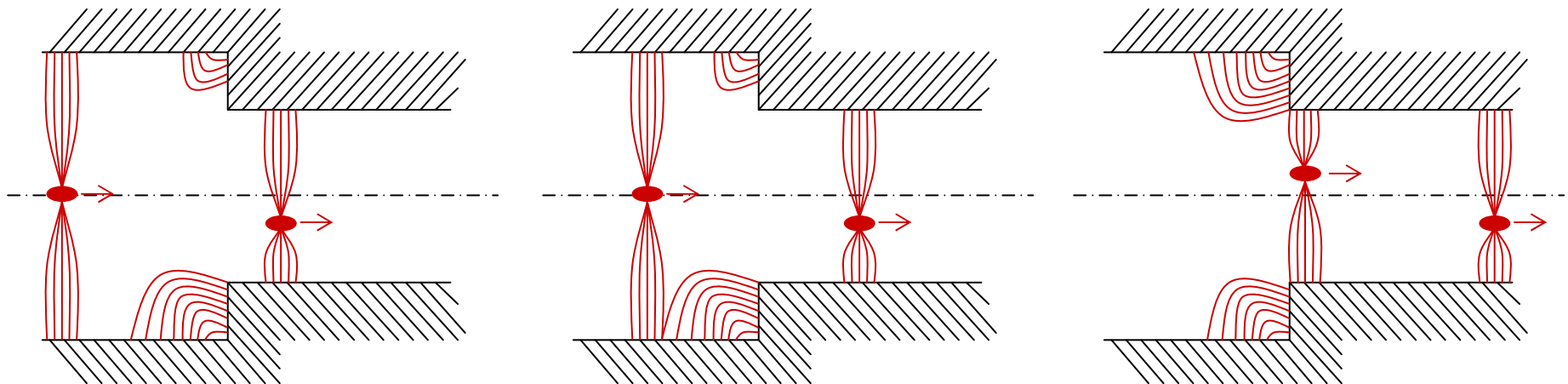


- Vacuum chamber wake fields generated by beam particles, mainly affect trailing particles and in the case of ultra-relativistic beams can only affect trailing particles.

Wake Fields and Instabilities



- Wake fields are transient effects, they are generated during the beam passage and then last for a finite amount of time that depends on the particular wake and on the geometry of the vacuum chamber.
- If the wake field lasts for the duration of a bunch (hundreds of ps typically), particles in the bunch tail can interact with the wakes due to the particles in the head and **single bunch instabilities** can be triggered (distortion of the longitudinal distribution, bunch lengthening, ...).

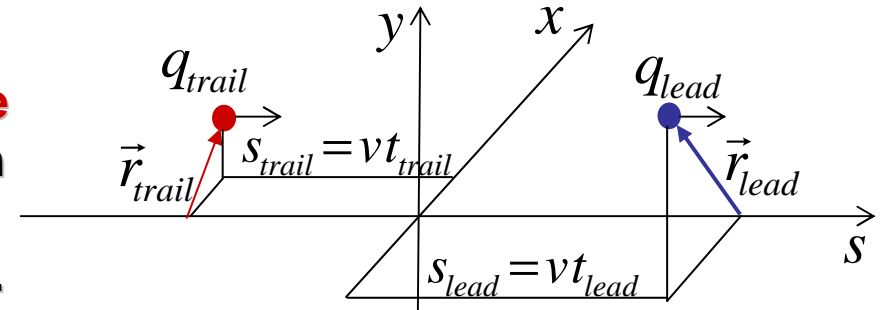


- If the wake field lasts longer, for example for the distance in time between bunches (several ns typically), wakes from leading bunches can interact with following bunches and potentially generate **multi-bunch or coupled bunch instabilities**.

Wake Potentials



- Wake fields effects can be divided into longitudinal and transversal. In the longitudinal case the wakes affect the energy of the particles, while in the transverse case is their transverse momentum to be affected.
- For this reason, in investigating longitudinal wake fields we consider *only the electric component of the wake fields*.
- It is often convenient to deal with wake potentials instead of wake fields. The **wake potential** is defined as the energy variation induced by the wake field of the leading particle on the unit charge trailing particle.



$$V_W(\vec{r}_{lead}, \vec{r}_{trail}, t_{trail} - t_{lead}) = \int_{-\infty}^{\infty} \vec{E}_W(s, \vec{r}_{lead}, \vec{r}_{trail}, t_{trail} - t_{lead}) \cdot d\vec{s},$$

- In practical cases, wake potentials are integrated over some finite length. The length of the integration path must be longer than the wake field maximum extension.
- In our results, $s = vt$ with v constant, this is a very good approximation for relativistic particles but it is also a reasonable assumption for the cases where the wake induced energy variation is small respect to the particle energy ⁹

Wake Functions



- The **wake function** is instead defined as the energy variation induced by the wake field of a unit charge leading particle on the unit charge trailing particle.

$$W(\vec{r}_{lead}, \vec{r}_{trail}, t_{trail} - t_{lead}) = \frac{V_W(\vec{r}_{lead}, \vec{r}_{trail}, t_{trail} - t_{lead})}{q_{lead}}$$

- The **total wake potential** for a bunch with charge distribution i with:

$$\int i(\vec{r}, t) d\vec{r} dt = Nq$$

is given by:

$$V(\vec{r}_{trail}, t_{trail}) = \int W(\vec{r}, \vec{r}_{trail}, t_{trail} - t) i(\vec{r}, t) d\vec{r} dt$$

- The total wake potential gives the energy variation that the trailing particle experiences due to the wakes of the whole bunch.
- Very often in real accelerators, we deal with distributions that “live” in the neighbor of the bunch center. In this case, it is sufficient to use the wakes on axis that can be obtained by setting r and $r_{trail} = 0$ in the previous expressions (**monopole wake approximation**).

Coupling Impedance



- The wake function represents the interaction of the beam with the external environment in the *time domain*.
- As for other phenomena, the equivalent *frequency domain* analysis can be very useful giving a different insight and additional interpretations.
 - The frequency domain “alter ego” of the wake function is the **coupling impedance**, measured in Ohm and defined as the *Fourier transform of the wake function*:

$$Z(\vec{r}, \vec{r}_{trail}, \omega) = \int_{-\infty}^{\infty} W(\vec{r}, \vec{r}_{trail}, \tau) e^{-j\omega\tau} d\tau \quad \text{with } \tau = t_{trail} - t$$

- If I is the Fourier transform of the charge distribution, the Fourier transform of the total induced voltage is simply given by:

$$\tilde{V}(\vec{r}, \vec{r}_{trail}, \omega) = Z(\vec{r}, \vec{r}_{trail}, \omega) I(\vec{r}, \omega)$$

- And the time domain expression can be obtained by the inverse Fourier transform:

$$V(\vec{r}, \vec{r}_{trail}, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{V}(\vec{r}, \vec{r}_{trail}, \omega) e^{j\omega\tau} d\omega$$

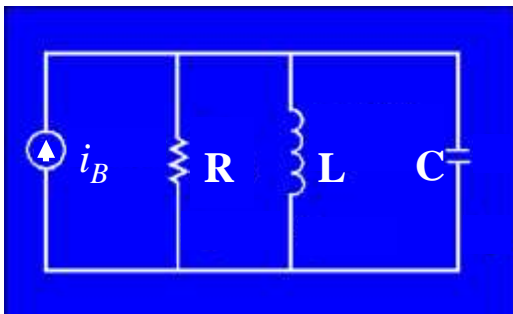
Interpretation of the Coupling Impedance



- The coupling impedance is a complex quantity with real and imaginary parts:

$$Z(\vec{r}, \vec{r}_{trail}, \omega) = Z_R(\vec{r}, \vec{r}_{trail}, \omega) + j Z_j(\vec{r}, \vec{r}_{trail}, \omega)$$

- There is a strong analogy between wake field and electronic circuit theories. This can be exploited and wakes can be represented by equivalent circuits.
- Analogously to the circuit case, the resistive part of the coupling impedance is responsible for the beam losses, while the imaginary part defines the phase relation between the beam excitation and the wake potential.
- For example, the impedance of a parallel RLC circuit is often associated to the impedance of the so-called *high order modes* (HOM), single resonance wakes in the vacuum chamber.



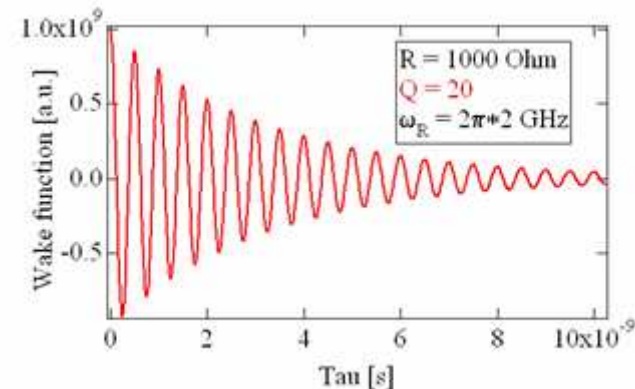
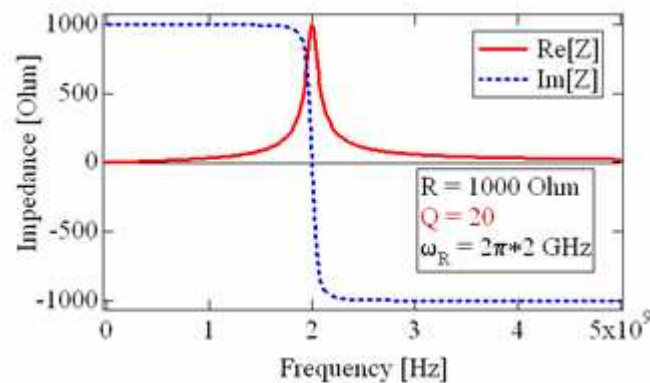
$$Z(\omega) = \frac{R}{1 + jQ \left(\frac{\omega}{\omega_R} - \frac{\omega_R}{\omega} \right)}, \quad \omega_R = \frac{1}{\sqrt{LC}}, \quad Q = R \sqrt{\frac{C}{L}}$$

$$W(\tau) = \begin{cases} 0 & \tau < 0 \\ \frac{e^{-\omega_R \tau / 2Q}}{C} \left[\cos\left(\omega_R \tau \sqrt{1 - 1/4Q^2}\right) - \frac{\sin\left(\omega_R \tau \sqrt{1 - 1/4Q^2}\right)}{\sqrt{4Q^2 - 1}} \right] & \tau > 0 \end{cases} \quad 12$$

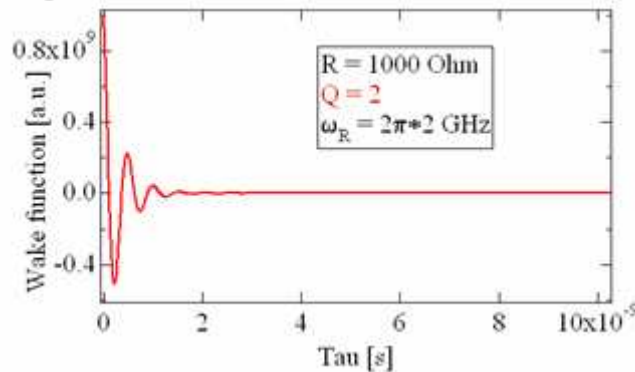
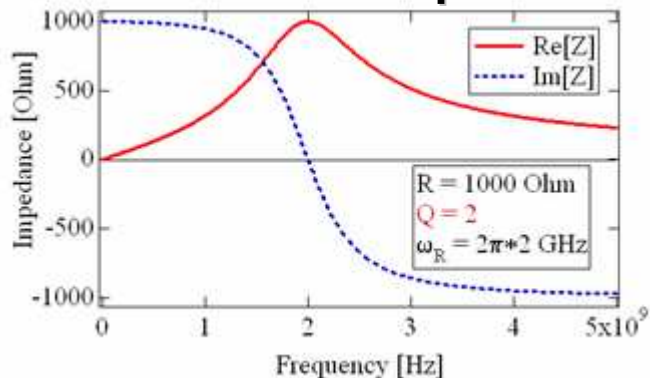
Narrow-band and Broad-band Coupling Impedances



- Using this RLC model the HOMs can be classified in two main categories.
- **Narrow-band impedances.** These modes are characterized by relatively high Q and their spectrum is narrow. The associated wake last for a relatively long time making this modes important for multibunch instabilities.



- **Broad-band impedances.** These modes are characterized by a low Q and their spectrum is broader. The associated wake last for a relatively short time making this modes important only for single bunch instabilities.



Transverse Wake Fields Case



- A similar approach and definitions can be used for the transverse wake case.
- The **transverse wake function** defines the *transverse momentum kick per unit leading charge and unit trailing charge* due to the wake fields.
- Transverse wake fields are excited when the beam passes out of center. If the displacement is small enough only the *dipole* term proportional to the displacement is important. In such a situation, the *transverse dipole wake function*, defined as the transverse wake function for unit displacement, can be used.
- The **transverse coupling impedance** is defined as the Fourier transform of the transverse wake function times j .
- Longitudinal and transverse wakes are representation of the same 3D wake field and are linked each other by the Maxwell equations.
The so-called **Panofsky-Wenzel relations** allow to calculate one wake component when the other is known.

Impedance of Accelerators



- In a real accelerators, the vacuum chamber has a very complex shape and includes many components that can potentially have “trapped” HOM.
- Anyway, not all the wakes excited by the beam can be trapped in the vacuum chamber. In fact, for a given vacuum chamber geometry, it exist a **cutoff frequency** such that modes with frequency above cutoff propagates along the chamber:

$$f_{Cutoff} \approx \frac{c}{a} \quad a \equiv \text{chamber transverse size}$$

- In summary, when the beam transits along the vacuum chamber it excites wake fields. These can be classified in three main categories:
 - wake fields that travels with the beam (such as the space charge);
 - wake fields that are localized in some resonant structure in the vacuum chamber (narrow and broad band HOM);
 - high frequency wakes, above the vacuum chamber cutoff, that propagates along the vacuum chamber. This last category does not generate any net interaction with the beam unless they are synchronous with the beam itself.

Impedance of Accelerators

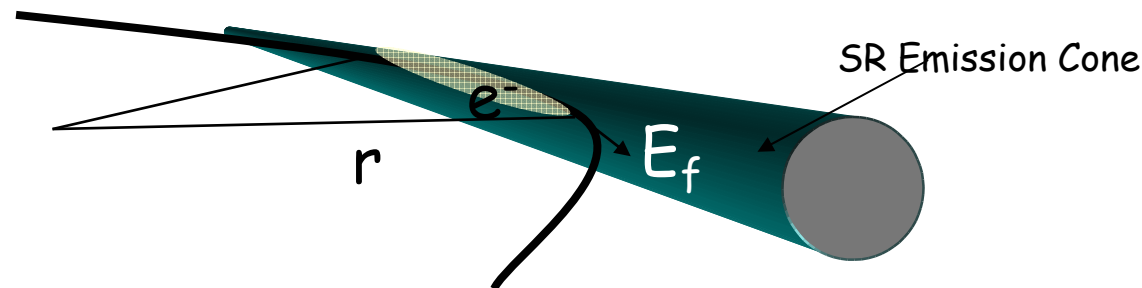


- **Narrow band impedances**, such as the ones due to the RF cavities and potentially to other components such as kickers and similar structures, are well localized and are usually treated independently.
- In the case of broad band impedances, the contributions of the single components (bellows, vacuum ports, transitions, diagnostics, ...) are individually calculated (mainly numerically) and/or measured and then added together.
- Most of the circular accelerators with relatively long bunches (greater than few tens of ps rms) present a total coupling impedance which is mainly inductive. Scaled models, such as the *broadband resonator* (parallel RLC with $Q \sim 1$ and $2\pi\omega_R \sim f_{Cutoff}$) or such as the empirical “*SPEAR scaling*” model, are often used to represent with some success the total broadband impedance of such accelerators.
- Concerning the wake fields that propagates with the beam, we already glanced on the space charge “wake”. Another example of such a kind of wake is the one represented by the *resistive wall impedance*, where the finite resistivity of the vacuum chamber walls generates a wake that can be often expressed as the impedance of a high-frequency broad-band resonator.

The Synchrotron Radiation “Wake” Field



- The wake field due to synchrotron radiation, belongs to the category of the wakes that propagates with the beam.
- Such a wake is important only for the relativistic particle case.
- Relativistic particles on a curved trajectory emit synchrotron radiation (SR). The SR fields propagates in a cone of emission centered on the tangent to the beam trajectory at the emission point and with $\sim 1/\gamma$ aperture.



- The fields propagate at the speed of light, while the particles move on the curved trajectory. For this reason, even if the particles are relativistic the projection of their speed on the tangent direction is smaller than c .
 - In other words, the SR wake field due to a particle in the tail of the bunch can reach and interact with a particle in the head!
- This is exact the opposite of what happens with vacuum chamber wakes.

Single Bunch Effects



Broadband impedances have important effects on accelerators.

In electron storage rings in the presence of radiation damping, the equilibrium distributions at low current are usually gaussian. By increasing the current per bunch, the wakes become stronger and can generate non gaussian equilibrium distributions.

In linacs and in heavy particle accelerators, broad band impedances can generate emittance and energy spread growth.

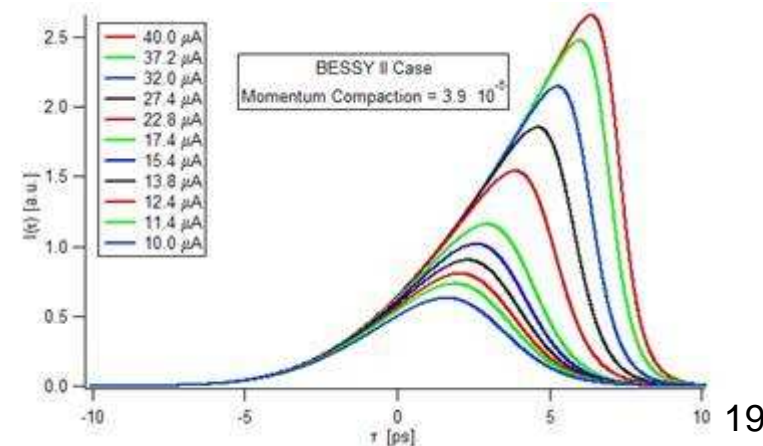
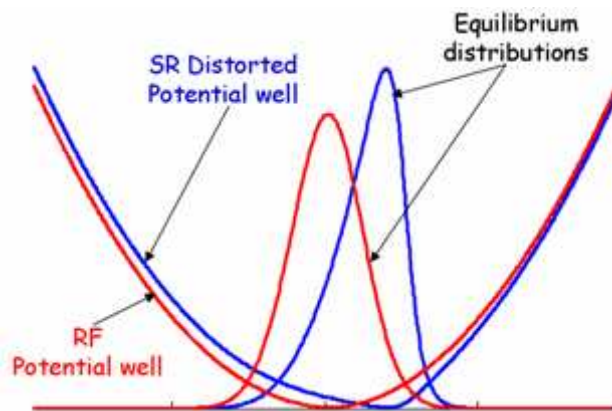
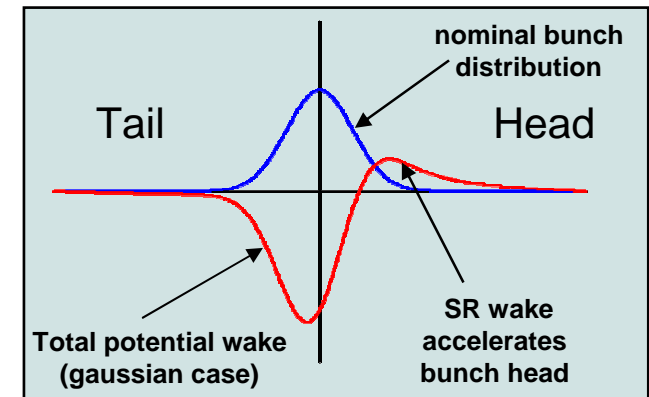
In all accelerators, if the current per bunch is increased further, the wakes can become strong enough to generate single bunch instabilities that can severely change the characteristics of the bunch and/or generate particle losses.

In what follows, some examples (not a complete review!) of such cases will be given.

Potential Well Distortion: The SR Wake Case



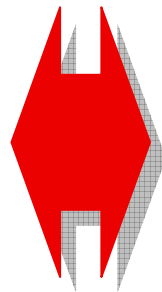
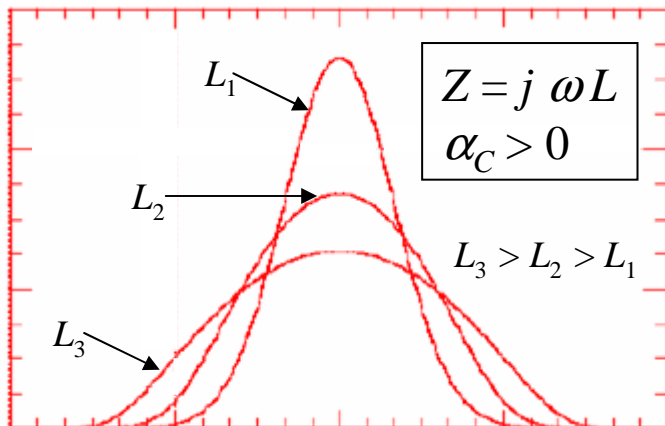
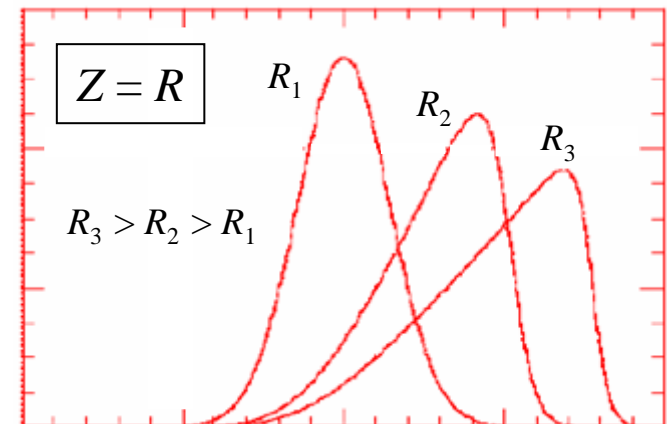
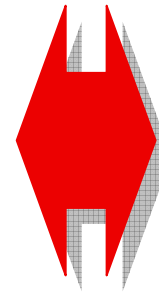
- The example concerns the case of the synchrotron radiation (SR) wake in electron storage rings.
- When a storage ring is tuned for short bunches (\sim few ps rms), the SR becomes the dominant wake.
- The SR wake due to the particles in the tail of the bunch interacts with particles in the head by changing their energy.
- This generates a distortion of the normally parabolic RF potential (**potential well distortion**). In this situation, the bunch is forced to a new equilibrium with a non-gaussian longitudinal distribution.



Potential Well Distortion: The General Case



- The potential well distortion mechanism, shown for the case of the SR wake, is actually quite general and common to all kind of wakes in electron rings.
- Remembering that wakes can be represented by the real and imaginary part of the coupling impedance, some common “rules” can be derived.
- The **real (resistive) part** of the coupling impedance generates *asymmetric distortions* and *lengthening* of the bunch distribution. The *bunch center of mass moves towards a different RF phase* to compensate for the wake induced energy losses.

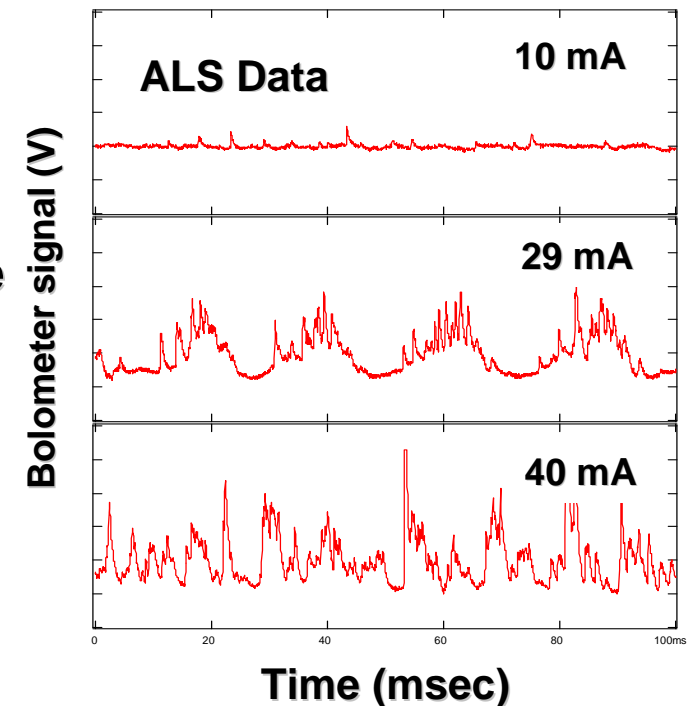
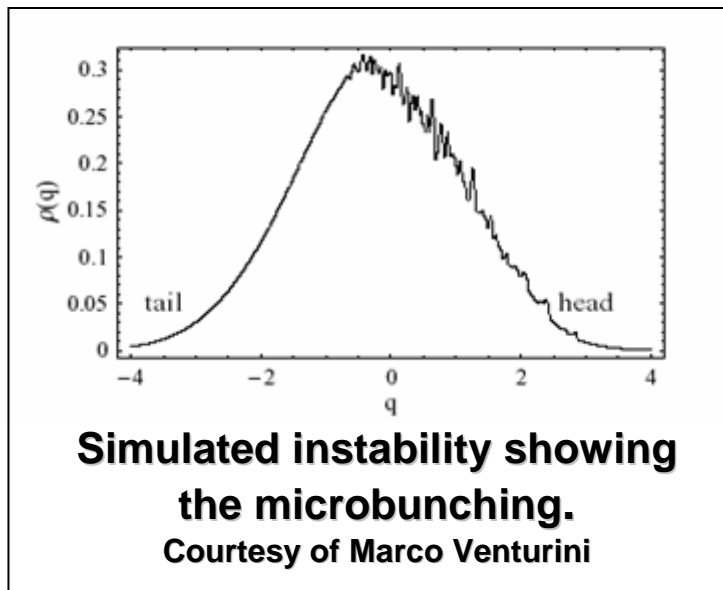


- The **imaginary (reactive) part** of the coupling impedance generates *symmetric distortions* of the bunch distribution. The *bunch center of mass does not move* (no energy losses). It generates *bunch lengthening or shortening*.

Single Bunch Instabilities: The SR Wake Case



- In an electron storage ring, if the current per bunch is above a specific threshold, the SR wake can drive a **microbunching instability** in the electron bunch.
- The SR wake becomes strong enough to create temporary micro-structures in the bunch that radiates strong “bursts” of coherent synchrotron radiation in the far-infrared.



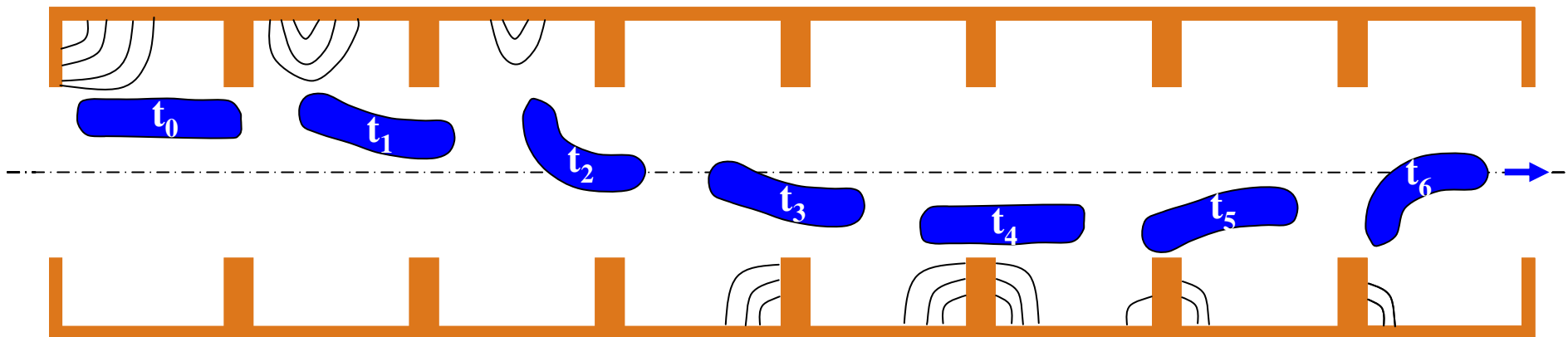
$$I_b > A \frac{1}{h^{1/2} f_0 \left(\hat{V}_{RF} \cos \varphi_s \right)^{1/2}} \frac{\alpha_C^{3/2}}{\rho^{11/6} J_s^{3/2}} \frac{\gamma^{9/2}}{\lambda^{2/3}}$$

$$A = \left(m_0^{1/2} e^{1/2} c^3 C_q^{3/2} \right) / \left(2\pi^{1/3} r_0 \right) \quad [MKS \text{ Units}]$$

Single Bunch Instabilities: Beam Break Up



- When a bunch enters off-axis in a linac structure it excites transverse wakes.
- If the impedance associated with the wake is broad-band, the head of the bunch can excite the wakes that will deflect the tail of the bunch.
- In long high current/bunch linacs the effect can build up and the bunch can be distorted into a “banana” like shape. This effect is known as **single-bunch beam break up (SBBU)**.



- The effect was first observed in 1966 at SLAC in the 2 miles long linac of the SLC (Stanford Linear Collider) and was responsible for luminosity limitation.

Single Bunch Instabilities: Microwave Instability



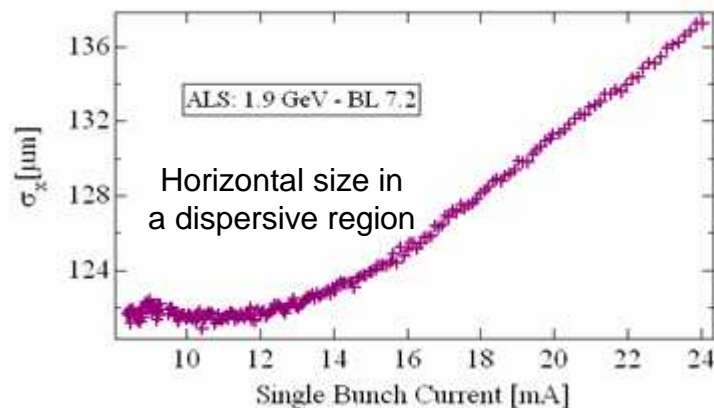
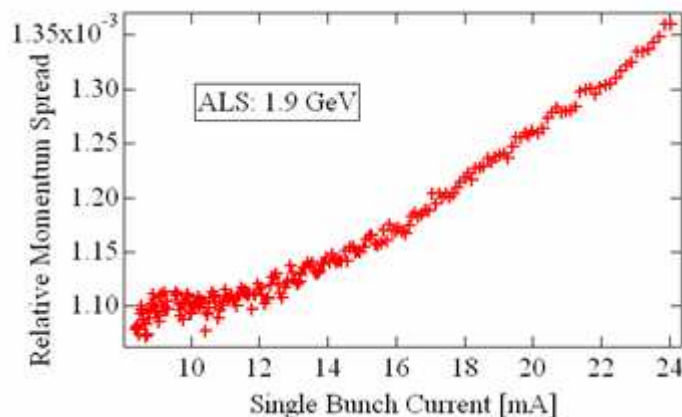
- The total broad band impedance of a storage ring is also responsible of another longitudinal single bunch instability known as the **microwave instability**.

- When the current per bunch is larger than the instability threshold:

$$I_{peak} > \frac{2\pi\alpha_C E_0 (\sigma_E / E_0)^2}{e|Z_{||}/n|}$$

the single particles get excited by the wakes on exponentially growing longitudinal oscillations. Because non-linearities, the oscillation frequency changes with amplitude limiting the maximum amplitude and in most of the cases no particle loss happens.

- The net effect on the bunch is an *increase of the energy spread* above threshold with a consequent *increase of the bunch length* and of the beam *transverse size in dispersive regions*.



Multi Bunch Instabilities

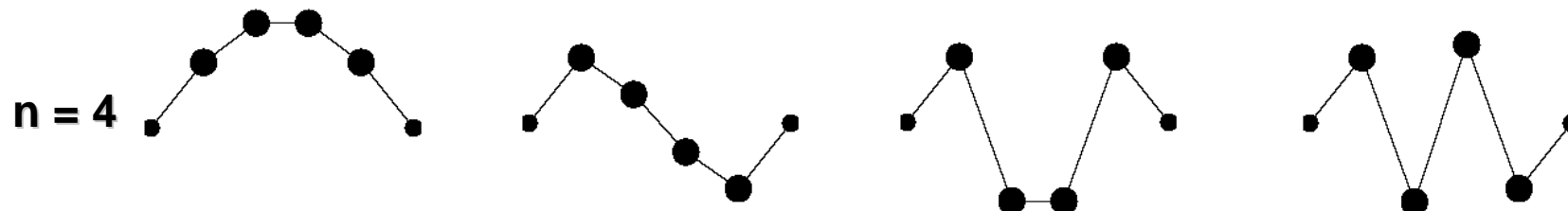
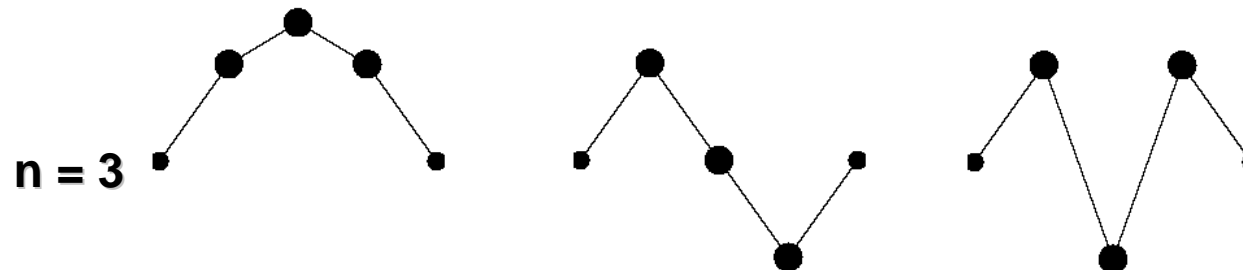
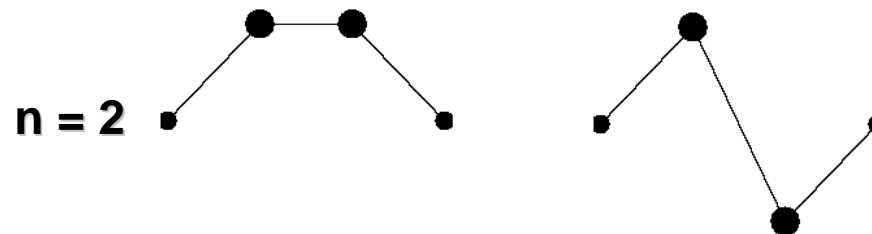
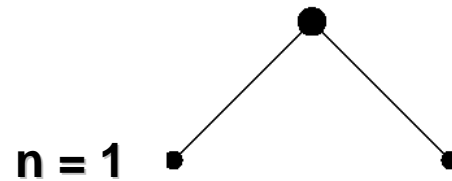


- In the case of narrow-band impedances the wake generated by one bunch can last long enough to interfere with other bunches or with the bunch itself in subsequent turns. In this situation multi-bunch instabilities can be excited.
- High current accelerators are carefully designed in order to minimize broad band and narrow band impedances. Anyway, even in the best conceived accelerator, the impedance cannot vanish and there will be always a current threshold above which the beam will become unstable.
If the accelerator is required to operate above the instability threshold, *active feedback systems* are necessary for damping down the instabilities.
- Despite these difficulties, properly designed accelerators with low overall broad-band impedance, carefully damped HOMs and active longitudinal and transverse bunch by bunch feedbacks achieved very remarkable results. Currents of few Amps have been stored in electron and positron machines (PEP 2, KEK-B, DAΦNE, ...) and of many tens of mA in proton machines (SPS, TEVATRON, HERA, ...).

Coupled Bunches Modes



From Dan Russell's
Multiple DOF Systems



Multi-Bunch Instabilities Mechanism



- By using the model of coupled harmonic oscillators, every mode can be characterized by a complex frequency ω and by the equation of a damped oscillator:

$$\varphi_n(t) = \hat{\varphi}_n e^{-(\text{Im}[\omega_n] + \alpha_D)t} \sin(\text{Re}[\omega_n]t + \varphi_{n0}) \quad \alpha_D \equiv \text{radiation damping}$$

- The oscillation becomes unstable (anti-damping) when:

$$\text{Im}[\omega] + \alpha_D < 0 \quad (\alpha_D > 0 \text{ always})$$

- Wakes fields produce a shift of the imaginary part of the frequency:

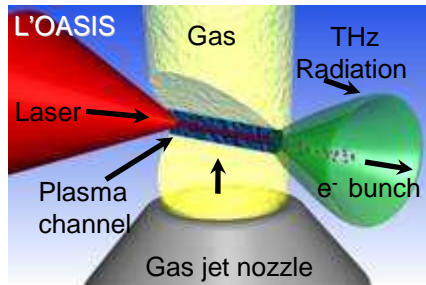
$$\Delta \text{Im}[\omega_n] \approx I_B \frac{e\alpha_C}{v_s E} Z(\omega_n)$$

- Depending on the signs of the momentum compaction and of the impedance, some modes can become unstable when the current per bunch is increased.
- Feedback systems increase α_D so that to increase the threshold for the instabilities.

“Good” Wake Fields

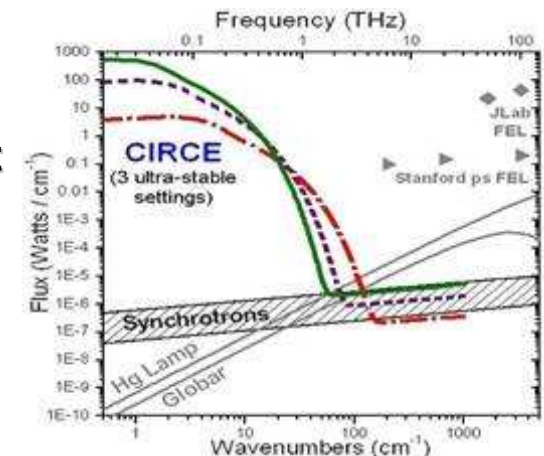


- Not all of these “evanescent and ghostly” wakes are bad in accelerator applications. In fact, there are few examples where wakes play a positive role:



- Wakefield-based acceleration schemes. Strong R&D and very promising results.

- Bunches in electron storage rings with longitudinal distribution asymmetrically distorted by wake-fields emit coherent synchrotron radiation at much higher frequencies than bunches with nominal gaussian distribution. This can be exploited for designing far-infrared synchrotron light sources with revolutionary performances.



Bad Wake

- Wake fields are commonly exploited in diagnostic systems used for the characterization of the beam properties.



God Wakes²⁷

References



L. Palumbo, V. G. Vaccaro, M. Zobov, “Wake fields and impedances”, CERN-95-06

A. Chao, “Physics of Collective Beam Instabilities in High Energy Accelerators”, Wiley-Interscience Pub. (1993).

A. Chao, M. Tigner, “Handbook of Accelerator Physics and Engineering”, Word Scientific Pub. (1998).

Possible Homework



- **Calculate, the repulsive force that a proton with a $100\text{ }\mu\text{m}$ displacement from the beam center will experience due to the space charge from the other protons in the beam. The beam has a circular profile with rms size of 2 mm and an energy of 2 GeV. The linear charge density is of 0.7 nC/m. Estimate if the effect on the particle integrated over one turn is significant or not (the ring length is 100 m). Compare with the case of an electron beam with the same characteristics.**
- **In principle, a particle accelerator built in the space (orbiting around the earth for example) could be built without a vacuum chamber. Will the particles in such an accelerator be subjected to any wake field? Please explain your answer.**